Technical Comments

Comment on "Theory of Thin Airfoils in Magnetoaerodynamics"

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IN a paper[1] examining the motion of a compressible fluid with arbitrary finite electrical conductivity in the presence of a thin airfoil, the author derived the equations

$$(1 - M^2)\partial u/\partial x + \partial v/\partial y = M^2\xi/A^2$$

$$\partial v/\partial x - \partial u/\partial y = \omega$$

where ξ and ω represent, respectively, the "curl" of the magnetic and velocity fields. The solution is taken in the form of the general solution of the homogeneous system added to a particular solution of the nonhomogeneous system. This procedure leads to three equations (4.4, 5.4, and 6.12) for two functions.

These three equations are inconsistent. The error occurs in that ξ and ω are not independent functions but are related to u and v through the magnetoaerodynamic equations (2.4–2.8).

Reference

[1] Dragos, L., "Theory of thin airfoils in magnetoaerodynamics," AIAA J. 2, 1223–1229 (1964).

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Reply by Author to P. Greenberg

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FOLLOWING the objection of P. Greenberg that in our paper[1] we have obtained three equations (4.4, 5.4, 6.12) for two functions and since these equations are inconsistent, we present a new solution of the problem—based essentially on the idea of the first paper—which shows that the error indicated does not exist.

It is shown in particular that the two functions φ and ψ must vanish. Accordingly the three equations (4.4, 5.4, 6.12) can exist. Besides, if in Ref. 1 the complete reasoning had been made, it would have resulted that the two functions must vanish.

Under the conditions as in Ref. 1 and using almost the same notations we have to integrate the following system,

$$M^2(\partial p/\partial x) + \text{div}\mathbf{v} = 0 \tag{1}$$

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$$\partial\mathbf{v}/\partial x = -\text{grad}p + (1/A^2)\mathbf{J} \times \mathbf{e}_2 \tag{2}$$

$$\text{rot}\mathbf{h} = \mathbf{J} = J\mathbf{e}_3 = R_M(\mathbf{E}_1 + \mathbf{e}_1 \times \mathbf{h} + \mathbf{v} \times \mathbf{e}_2) \tag{3}$$

$$\text{rot}\mathbf{E}_1 = 0 \tag{4}$$

$$\text{div}\mathbf{h} = 0 \tag{5}$$

Since the motion is a plane one from (3) and (4), there results $\mathbf{E}_1 = 0$.

From (3) we deduce the equation of magnetic induction

$$R_M^{-1}\,\Delta\mathbf{h} = \frac{\partial\mathbf{h}}{\partial x} - \frac{\partial\mathbf{v}}{\partial y} + \mathbf{e}_2\,\text{div}\mathbf{v} \qquad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{6}$$

By applying the operator rot in Eq. (2) we get

$$\partial\omega/\partial x = (1/A^2)(\partial J/\partial y) \tag{7}$$

By derivation with respect to the variable x of Eq. (2) and using Eq. (1), we eliminate the pressure. By applying then the operator div to the equation found, we have

$$H\theta = \left[(1 - M^2)\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right]\theta = \frac{M^2}{A^2}\frac{\partial^2 J}{\partial x^2} \tag{8}$$

Finally, applying the operator rot in Eq. (6) we obtain

$$PJ = \left(R_M^{-1}\Delta - \frac{\partial}{\partial x}\right)J = -\frac{\partial\omega}{\partial y} + \frac{\partial\theta}{\partial x} \tag{9}$$

We have used the following notations:

$$\text{div}\mathbf{v} = \theta \qquad \text{rot}\mathbf{v} = \omega\mathbf{e}_3 \tag{10}$$

From Eqs. (7–9) we have

$$T\theta = T\omega = TJ = 0 \tag{11}$$

where T is the operator

$$T = N^{-1}\Delta H\frac{\partial}{\partial x} + (A^2M^2 - A^2 - M^2)\frac{\partial^4}{\partial x^4} - (A^2 + M^2 - 1)\frac{\partial^4}{\partial x^2\partial y^2} + \frac{\partial^4}{\partial y^4} \tag{12}$$

with the following notation

$$R_M = A^2N \tag{13}$$

N being the parameter of the magnetohydrodynamic interaction.

The fact that the three unknowns θ, ω, and J satisfy the same equation (11) is of a particular importance for determining the solution of the problem.

We shall look for the general solution of Eq. (11) under the form of a continuous superposition of plane waves of the form

$$\exp[-i\lambda(x + sy)] \tag{14}$$

By replacing in (11) and (12) we get

$$as^4 - bs^2 + c = 0 \tag{15}$$

where

$$\left.\begin{aligned} a &= 1 - (i\lambda/N) \\ b &= A^2 + M^2 - 1 - (i\lambda/N)(M^2 - 2) = b_1 - (i\lambda/N)b_2 \\ c &= A^2M^2 - A^2 - M^2 - (i\lambda/N)(1 - M^2) = c_1 - (i\lambda/N)c_2 \end{aligned}\right\} \tag{16}$$